Interdisciplinary Computer Analyses of Three-Dimensional Solids Defined by Polyhedral Surfaces

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Introduction

POLYHEDRAL representations of three-dimensional solids have been employed to advantage in computerized engineering analyses involving several disciplines. Because such a definition of a solid is relatively simple to construct and lends itself to numerical computations, the same basic definition can be used in each analysis. The resulting geometric compatibility, facilitates performing analyses where the geometric consequence of one solution influences the computation in a separate discipline.

Geometric Definitions

Polyhedrons can be conveniently described by listing the coordinates of each corner or vertex and the manner in which these vertices connect to form faces. This format is consistent with that employed in plotting routines.[1] The order of the points in the face strings indicates the direction of the solid with respect to the face. Actual computer card input formats for vertex coordinates and face lists are shown in Fig. 1. In this format E is a flag to signify the end of the vertex or face card sets. It is zero except on the last vertex card and the last face string card, where it is set equal to 1. NV and NF are the vertex and face string numbers, respectively. Face string entries NV_i represent the numbers of the vertices in the order that they would be encountered if one were to stand on the face and walk around the perimeter in such a manner that the left foot stays on the edge and the right one on the interior of the face. Faces with multiple perimeters have each perimeter list input separately with the same face number assigned. All face lists terminate with the same vertex number that they start with (closed lists) as illustrated in the data for a unit cube shown in Fig. 1.

When the solid contains a large number of faces it is convenient to employ automatic data generation techniques. This aspect has been simplified through use of a digitizer interfaced to an IBM 026 keypunch machine. Scale drawings of the parts are traced with the digitizer stylus, which produces appropriate keypunch cards containing the coordinates of each vertex. Face lists are generated by a computer subroutine which connects vertices in multidimensional arrays. A series of digitizer routines have been implemented to facilitate such operations as interpolating points along circular arcs, generating mirror images, or scaling and translating cross section definitions.

Engineering Calculations

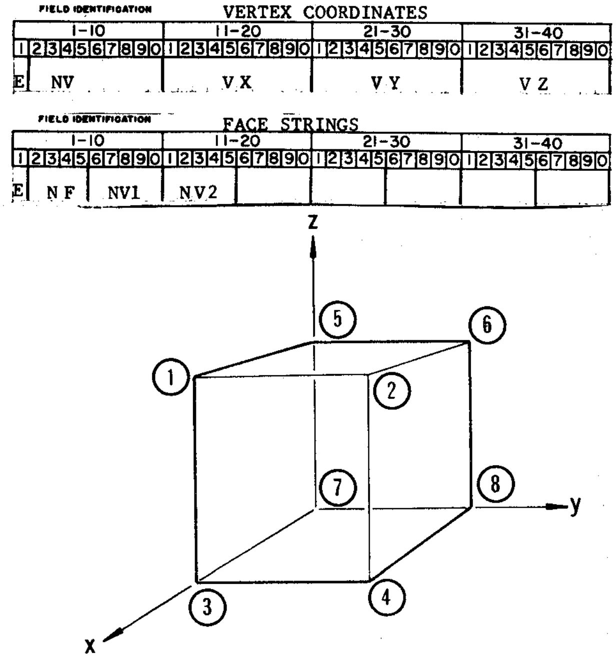
Polyhedral surface definitions are well adapted to a variety of analytical computations, although familiar forms of the equations may need to be converted into surface formats through mathematical manipulations. Among the mathematical tools employed in surface analyses, the two most often applied are the Gauss divergence theorem, and quadrature integration over surfaces. Gauss' divergence theorem translates a volume integral into a surface integral and is applicable whenever the function can be expressed as the divergence of a vector point function. Such a transformation arises in computing the moment of inertia of a solid.

$$Ixx = \int\int\int \rho[y^2+z^2]\, dV$$

Here ρ is the density and x, y, and z are Cartesian coordinates. An equivalent surface integral expression is

$$Ixx = \rho \int\int [yz^2 n_y + zy^2 n_z]\, dS$$

where n_y and n_z are y and z components of a unit vector normal to the surface. Quadrature rules can be applied to perform the integration. A four point Gauss-Radau rule was employed for mass property calculations which include centroid, volume, moments and products of inertia in a single computer subroutine.[2]



	E						
1		8	6	NO. OF VERTICES AND NO. OF FACES			
2		1		1.0		0.0	1.0
3		2		1.0		1.0	1.0
4		3		1.0		0.0	0.0
5		4		1.0		1.0	0.0
6		5		0.0		0.0	1.0
7		6		0.0		1.0	1.0
8		7		0.0		0.0	0.0
9	1	8		0.0		1.0	0.0
10		1	1	2	4	3	1
11		2	2	6	8	4	2
12		3	5	6	2	1	5
13		4	6	5	7	8	6
14		5	1	3	7	5	1
15	1	6	3	4	8	7	3

Fig. 1 Input format and data for unit cube.

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Shell Analyses

The data structure employed in the polyhedron definitions is directly compatible with finite element shell stress analyses programs employing quadrilateral plate elements. These elements have been found to yield stable results in a variety of static and dynamic analyses. The particular programs employed in conjunction with surface analyses were SNAP and SNAP dynamics.[3] These programs require a nodal coordinate definition and a separate element connectivity list analogous to the polyhedron face string lists.

Three-Dimensional Solid Stress Analyses

Surface integral methods for computer calculation of stresses in a solid have received renewed attention in recent years,[4] and one such program[5] was acquired and adapted to this format. This program already contained the vertex and face string data structure, and thus only minor alterations to format were required to interface it with the polyhedron analyses system.

Unlike differential equation formulations, which are subsequently discretized at fixed interconnected locations spanning the volume occupied by the solid, integral formulations cover only the surface. Both methods involve the solution of large sets of simultaneous equations, which are treated as linear matrices. The major difference between the two methods is in the form of the matrices. Finite element techniques produce large, sparse, banded systems, while integral methods yield smaller, full matrices.

Propellant Grain Burn Surface Area

Computation of the burning surface geometry of a solid propellant grain (fuel charge) can be a complex graphic problem. The surface burns back parallel to itself until completely consumed. The exposed burning surface at any instant determines chamber pressure and motor thrust, which establish a variety of structural loads. Mathematically this problem is analogous to wave front analysis or to the cutter tool offset problem in numerically controlled machining of a surface. A program was developed to solve this particular problem, using the surface definition data. Basis for this routine was a shortest path (ray) evaluation. Each point on a burnout (or termination) surface is examined in turn to establish the closest point on an initial burn surface, and burn prisms are computed. Cross section areas along each such prism are summed to yield the total solution.

Computer times for this three-dimensional burn area analysis are determined primarily by the closest point calculation algorithm. Typical run times for grains with 100 surface elements are 6 min on an IBM 360-40 computer. Grain configurations used as input are shown in Fig. 2.

Combined Analyses

Although automatic sequencing of these analyses is possible due to the common data structure, it has been necessary to evaluate the solution from one analysis before running the next. Propellant grain stresses, burnback areas, and mass properties

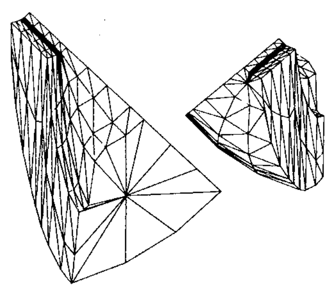

Fig. 2 Propellant grain sector represented as a polyhedron for computer analysis.

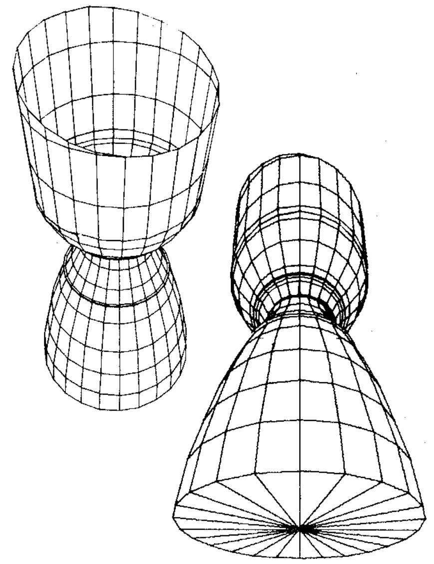

Fig. 3 Views of submerged portion of space shuttle booster during water entry.

have, however, been calculated from a common input. Structural deformations of the propellant grain due to thermal effects are often large enough to influence the burn areas, and a combination of these analyses would be relatively simple to perform. Cost of the three-dimensional stress analyses, however, is currently too high to permit this application in routine calculations of this type.

Combinations of these various analyses have been required for several problems, one of which is shown in Fig. 3. This particular problem has been the object of renewed interest in conjunction with the recovery of Space Shuttle components. Here the hydrostatic pressures encountered at some submergence depth for an expended booster motor are required as input to a shell analysis program. Buoyancy forces and moments are determined by the mass properties of the submerged portion, which are found by passing a sectioning plane through the polyhedron and computing the mass properties of the submerged portion. Pressures at each nodal point in punched cared format obtained from this analysis are then input directly to the finite element shell analysis.

Conclusions

Surface definitions have proven to be well suited to interdisciplinary computer operations and lend themselves easily to formulation of a variety of three-dimensional engineering problems. Integral formulations of boundary value problems that are directly compatible with this technique are, however, not as highly developed as the differential finite element and finite difference routines.

The polyhedron format employed in these applications is believed to be consistent with over-all practicality at this time. Extensions to surface integration over curved patches is straightforward, but plotting and geometric routines for such definitions are not common.

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Scattering Properties of Ice Particles Formed by Release of H_2O in Vacuum

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Introduction

LIQUID water droplets released in space during the Apollo missions quickly froze and formed luminous clouds of ice particles. The thermodynamics and optical properties of droplets formed in this manner have become a topic of current interest.[1,2] In the work reported herein, the scattering characteristics of an aggregate of ice particles formed by injecting a stream of water into a vacuum tank were determined experimentally. Although the experiments were performed in the absence of solar radiation, the laboratory freezing conditions are similar to those encountered during the Apollo flights. The measured optical properties are believed to be identical to those that would be observed from a cloud formed by release of a liquid from a spacecraft.

Experimental

A concentrated flow of ice particles was formed by introducing a stream of water through a 0.4-mm orifice into an evacuated 325 ft^3 vacuum tank. The stream broke into individual particles while cooling. After they had frozen, the particles were collected and collimated by means of a 4-ft-mylar funnel placed approximately 10 ft from the water injection orifice. The funnel produces a vertical flow of frozen particles. The optical arrangement used for scattering measurements is shown in Fig. 1. Ice particles flow along the z axis. A white light source produces a parallel beam that rotates in the xy plane around the particle flow at a

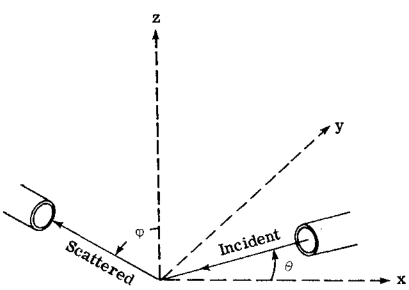

Fig. 1 Optical arrangement for scattering experiments; ice particles flow along the z axis.

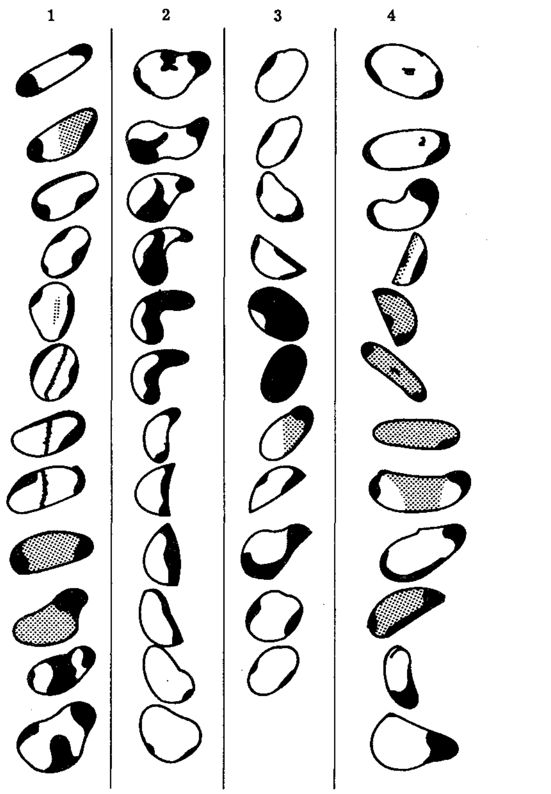

Fig. 2 Reproductions of four particles tumbling during free fall; dark areas corespond to bright spots on the particles.

speed of approximately one rps. The scattered radiation is recorded by a stationary 1P21 photomultiplier tube located in the xz plane and at an angle, ϕ, with respect to the z axis. The diameter of the ice flow was 2.5 cm, approximately twice as large as the light beam. High speed (4000 frames/sec) motion pictures were taken of the ice flow, and conventional photographs obtained at a shutter speed of 0.002 sec provided particle size distribution data. The absolute quantity of water evaporated during the cooling process was determined by attaching a calibrated glass tube, cooled by liquid nitrogen, to the base of the funnel. After collecting the ice particles in the tube, air was reintroduced into the vacuum tank and the ice particles were allowed to melt. By comparing the volume of water introduced into the tank with the collected volume, it was determined that approximately 80% of the water injected through 0.4- and 0.7-mm orifices evaporates during the cooling process. The experiments were repeated with a saturated solution of $CaCl_2$, but the droplets failed to freeze.

Results

The particle size distribution for a 0.4-mm orifice was obtained by measuring the size of each particle in a conventional photograph of the verticle flow. Particle size is defined here as the largest dimension of the particle.

Table 1 Ice particle size distribution for a 0.4 mm orifice

Particle size, mm	0–0.2	0.2–0.4	0.4–0.6	0.6–0.8	0.8–1.0	1.0–1.2
Fraction of total particles	0.27	0.26	0.21	0.17	0.07	0.02

High speed motion pictures of the flow, projected onto a large screen, made millimeter size particles appear to be 50 cm in

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